第一章　相对论原理

1.2. Calculate the relativistic masses and momenta of the electrons in the three cases in Problem 1.1.

Solution: The mass of an electron is $9.109\times10^{-31}kg$ .Hence,the rest energy of the electron is:

$$m_0c^2=\left(9.109\times10^{-31}kg\right)\left(2.9979\times10^{8}m/s\right)^2=8.187\times10^{-14}J$$
$$=\left(8.187\times10^{-14}J\right)\left(1eV/1.602\times10^{-19}J\right)=0.5110MeV$$

We obtain the total energy by adding the kinetic energy and the rest energy :

$$E=K+m_0c^2$$

According to the three cases in Problem 1.1:

$$K_1=100KeV=0.1MeV,K_2=1MeV,K_3=10MeV$$

Thus, $E_1=0.611MeV,E_2=1.511MeV,E_3=10.511MeV$

Using the equation about mass-energy equivalence together with the relativistic mass expression:

$$E=\frac{m_0c^2}{\sqrt{1-\frac{v^2}{c^2}}},m=\frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}},orE=mc^2$$

Hence,we get the relativistic masses are:

$$m_1=\frac{E_1}{c^2}=\frac{0.611MeV}{\left(2.9979\times10^{8}m/s\right)^2}=\frac{0.611\times10^{6}\times1.602\times10^{-19}J}{\left(2.9979\times10^{8}m/s\right)^2}=10.891\times10^{-31}kg$$
$$m_2=\frac{E_2}{c^2}=\frac{1.511MeV}{\left(2.9979\times10^{8}m/s\right)^2}=\frac{1.511\times10^{6}\times1.602\times10^{-19}J}{\left(2.9979\times10^{8}m/s\right)^2}=26.933\times10^{-31}kg$$
$$m_3=\frac{E_3}{c^2}=\frac{10.511MeV}{\left(2.9979\times10^{8}m/s\right)^2}=\frac{10.511\times10^{6}\times1.602\times10^{-19}J}{\left(2.9979\times10^{8}m/s\right)^2}=187.358\times10^{-31}kg$$

To calculate the momenta of the electrons,we can use the equation:

$$E^2=p^2c^2+m_0^2c^4$$

Hence,we get the momenta of the electrons are:

$$p_1 = \frac{\sqrt{E_1^2 - m_0^2 c^4}}{c} = \frac{\sqrt{0.611^2 - 0.511^2}MeV}{c} = 0.335MeV/c$$

$$p_2 = \frac{\sqrt{E_2^2 - m_0^2 c^4}}{c} = \frac{\sqrt{1.511^2 - 0.511^2}MeV}{c} = 1.422MeV\ /c$$

$$p_3 = \frac{\sqrt{E_3^2 - m_0^2 c^4}}{c} = \frac{\sqrt{10.511^2 - 0.511^2}MeV}{c} = 10.499MeV\ /c$$

1.6. A space shuttle moving in a vertical direction away from an observer on earth at a velocity of 0.2c fires a bullet with a velocity of 0.9c relative to the shuttle and in the same direction.What is the velocity of the bullet as seen by an observer on the ground?

Solution: The speed of the space shuttle relative to the stationary observer is $v = 0.2c$.The speed of the bullet in the frame of the reference of the shuttle is $u' = 0.9c$.Therefore,the speed u of the bullet relative to the stationary observer on the ground is

$$u = \frac{u' + v}{1 + \frac{u'v}{c^2}} = \frac{0.9c + 0.2c}{1 + \frac{(0.9c)(0.2c)}{c^2}} = 0.93c$$

Or

$$u' = \frac{u - v}{1 - \frac{uv}{c^2}} \rightarrow u = 0.93c$$

The bullet moves in a vertical direction away from an observer.

1.10.Calculate the velocity of a distant galaxy when a blue 434nm line is observed at 1950nm.Where does this line occur in the electron magnetic spectrum?

Solution: For electromagnetic radiation the formula in terms of wavelength becomes:

$$\lambda' = \lambda\sqrt{\frac{1+\beta}{1-\beta}}$$

Where, $\beta = \frac{v}{c}$

Hence,

$$\lambda' = \lambda\sqrt{\frac{1+\frac{v}{c}}{1-\frac{v}{c}}} \rightarrow \frac{\lambda'}{\lambda} = \sqrt{\frac{c+v}{c-v}} \rightarrow \left(\frac{1950}{434}\right)^2 = \frac{c+v}{c-v} \rightarrow v = \frac{\left(\frac{1950}{434}\right)^2 - 1}{\left(\frac{1950}{434}\right)^2 + 1}c = 0.9056c$$